

Fine-tuning and the Wilson renormalization group

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Abstract

We use the Wilson renormalization group (RG) formulation to solve the fine-tuning procedure needed in renormalization schemes breaking the gauge symmetry. To illustrate this method we systematically compute the non-invariant couplings of the ultraviolet action of the SU(2) pure Yang-Mills theory at one-loop order.

Keywords: Renormalization group formalism, gauge theory, Slavnov-Taylor identity

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1 Introduction

The Wilsonian renormalization group (RG) provides the most physical framework to study general properties of quantum field theories. In this formulation [1, 2] quantum fluctuations at short distances are integrated out to give an effective action for longer distances. In this approach the bare action can be thought as the result of integrating out the degrees of freedom with frequencies higher than the ultraviolet (UV) scale Λ_0 of the underlying theory (eventually more fundamental). By further integrating out the fields with frequencies up to some lower scale Λ one obtains the so called Wilsonian effective action. In this process one needs to introduce a scale which may conflict with symmetries. For a gauge theory, the gauge symmetry is completely broken at the intermediate scale Λ and one has to show that the physical theory (*i.e.* when all cutoffs are removed) satisfies the Slavnov-Taylor (ST) identity. This problem has been investigated in refs. [3]-[7] where it has been proved that by finely tuning the couplings in the Wilsonian effective action the usual ST identity is recovered at $\Lambda = 0$, at least in perturbation theory. This fine-tuning provides some of the boundary conditions of the RG flow. In particular the breaking of the symmetry can be studied at the physical point $\Lambda = 0$ [4] or at the UV scale [3, 7]. In the former case one enforces the relevant part of the usual ST identity and determines the boundary conditions at the physical point $\Lambda = 0$ of the relevant non-invariant couplings of the Wilsonian effective action in terms of physical vertices evaluated at the subtraction point. Due to the non-locality of these vertices, the fine-tuning is analytically difficult to solve (see ref. [4] where the boundary conditions of six relevant non-invariant couplings of a pure gauge $SU(2)$ theory at the first loop order were found in this way). In the latter the fine-tuning provides the couplings of the bare action, which contains all possible relevant interactions.

The conflict between symmetries and regularization is a long-standing problem in quantum field theory: in presence of chiral fermions no consistent regularization is known to preserve chiral symmetry and all possible non-invariant counterterms must be added to the classical action in order to compensate the breaking of the symmetry generated by the regularization. Although for non-anomalous theories this is an algebraic solvable problem, the determination of these counterterms is quite difficult in practice. In this paper we will use the RG formulation and the fine-tuning at the UV scale to provide all non-invariant counterterms of the bare action at the one-loop order. In order to keep things as simple as possible we will restrict to a pure gauge $SU(2)$ theory. The inclusion of chiral fermions can be done following the same lines [8, 7].

The paper is organized as follows. In section 2, after introducing some notation, we give a brief description of the RG method for the gauge case. In section 3 we recall the effective ST identity for this theory. In section 4 we determine the non-invariant couplings of the bare action at one-loop order by solving the fine-tuning at $\Lambda = \Lambda_0$. Technical details of this calculation are given in the appendix. In section 5 we present some concluding remarks and hints for the generalization of this computation to higher loops.

2 Renormalization group flow

Consider the $SU(2)$ gauge theory described by the classical Lagrangian (in the Feynman gauge)

$$S_{YM} = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{2} (\partial^\mu A_\mu)^2 - \bar{c} \cdot \partial^\mu D_\mu c \right\},$$

where the gauge stress tensor and the covariant derivative are given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g A_\mu \wedge A_\nu$, $D_\mu c = \partial_\mu c + g A_\mu \wedge c$ and we have introduced the usual scalar and external $SU(2)$ products. This action is invariant under the BRS transformation [9]

$$\delta A_\mu^a = \frac{1}{g} \eta D_\mu^{ab} c^b, \quad \delta c^a = -\frac{1}{2} \eta \epsilon^{abc} c^b c^c, \quad \delta \bar{c}^a = -\frac{1}{g} \eta \partial^\mu A_\mu^a$$

with η a Grassmann parameter.

Introducing the sources u_μ^a , v^a associated to the variations of A_μ^a , c^a respectively one has the BRS action [9]

$$S_{BRS}[\Phi, \gamma] = S_{YM} + \int d^4x \left\{ \frac{1}{g} u_\mu^a D_\mu^{ab} c^b - \frac{1}{2} \epsilon^{abc} v^a c^b c^c \right\} \quad (1)$$

where we have denoted by Φ and γ the fields and the BRS sources

$$\Phi_I = \{A_\mu^a, c^a, \bar{c}^a\}, \quad \gamma_i = \{w_\mu^a, v^a\},$$

and $w_\mu^a = u_\mu^a + g \partial_\mu \bar{c}^a$ (no source is introduced for \bar{c}^a).

According to Wilson [1, 2] the fields with frequencies $\Lambda^2 < p^2 < \Lambda_0^2$ are integrated in the path integral giving

$$Z[J, \gamma] = N[J; \Lambda, \Lambda_0] \int \mathcal{D}\Phi \exp i\left\{ -\frac{1}{2} (\Phi D^{-1} \Phi)_{0\Lambda} + (J\Phi)_{0\Lambda} + S_{\text{eff}}[\Phi, \gamma; \Lambda, \Lambda_0] \right\}, \quad (2)$$

where $N[J; \Lambda, \Lambda_0]$ contributes to the quadratic part of $Z[J, \gamma]$ and the functional S_{eff} is the Wilsonian effective action which is the generator of the connected amputated cutoff Green functions (except the tree-level two-point function). S_{eff} contains cutoff propagators $D_{\Lambda\Lambda_0}$ given by the free propagators $D(p)$ multiplied by a cutoff function $K_{\Lambda\Lambda_0}$ which is one for $\Lambda^2 \lesssim p^2 \lesssim \Lambda_0^2$ and rapidly vanishes outside. In (2) we have also introduced the cutoff scalar products between fields and sources

$$\begin{aligned} \frac{1}{2} (\Phi D^{-1} \Phi)_{\Lambda\Lambda_0} &\equiv \int_p K_{\Lambda\Lambda_0}^{-1}(p) p^2 \left\{ \frac{1}{2} A_\mu^a(-p) A_\mu^a(p) - \bar{c}^a(-p) c^a(p) \right\}, \quad \int_p \equiv \int \frac{d^4p}{(2\pi)^4}, \\ (J\Phi)_{\Lambda\Lambda_0} &\equiv \int_p K_{\Lambda\Lambda_0}^{-1}(p) \left\{ j_\mu^a(-p) A_\mu^a(p) + [\bar{\chi}^a(-p) - \frac{i}{g} p_\mu u_\mu^a(-p)] c^a(p) + \bar{c}^a(-p) \chi^a(p) \right\}. \end{aligned}$$

Notice that at $\Lambda = \Lambda_0$ the Wilsonian effective action coincides with the bare action.

The requirement that $Z[J, \gamma]$ is independent of the infrared cutoff Λ can be translated into a flow equation for S_{eff} , referred to as the exact RG equation. For our purposes it is convenient to perform a Legendre transform on S_{eff} in order to obtain the so-called ‘‘cutoff effective action’’ $\Gamma[\Phi, \gamma; \Lambda, \Lambda_0]$, which is the generator of 1PI cutoff vertex functions and

reduces to the physical quantum effective action in the limits $\Lambda \rightarrow 0$ and $\Lambda_0 \rightarrow \infty$. The flow equation can be written as [10]-[12]

$$\Lambda \partial_\Lambda \Pi[\Phi, \gamma; \Lambda, \Lambda_0] = -\frac{i}{2} \int_q \left[\frac{\Lambda \partial D_{\Lambda \Lambda_0}^{-1}(q)}{\partial \Lambda} \right]_{AB} (-1)^{\delta_A} \left[\frac{\delta^2 \Gamma[\Phi, \gamma; \Lambda, \Lambda_0]}{\delta \Phi_A(q) \delta \Phi_B(-q)} \right]^{-1}, \quad (3)$$

where $\delta_A = 1$ if Φ_A is a fermionic field and 0 otherwise and

$$\Pi[\Phi, \gamma; \Lambda, \Lambda_0] = \Gamma[\Phi, \gamma; \Lambda, \Lambda_0] + \frac{1}{2}(\Phi, D^{-1}\Phi)_{\Lambda \Lambda_0} - \frac{1}{2}(\Phi, D^{-1}\Phi)_{0 \Lambda_0}. \quad (4)$$

This equation can be integrated by giving boundary conditions in Λ . In order to set them we distinguish between relevant couplings and irrelevant vertices according to their mass dimension. Relevant couplings have non-negative mass dimension and are defined as the value of some vertices and their derivatives at a given normalization point (see [4] for their definition). For the $SU(2)$ theory there are nine relevant couplings which are the coefficients of the following monomials

$$\begin{aligned} & \frac{1}{2}A_\mu(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) \cdot A_\nu, \quad w_\mu \cdot \partial_\mu c, \quad \frac{1}{2}A_\mu \cdot A_\mu, \quad \frac{1}{2}(\partial_\mu A_\mu)^2, \\ & (\partial_\nu A_\mu) \cdot A_\mu \wedge A_\nu, \quad w_\mu \cdot c \wedge A_\mu, \quad \frac{1}{2}v \cdot c \wedge c, \\ & \frac{1}{4}(A_\mu \wedge A_\nu) \cdot (A_\mu \wedge A_\nu), \quad \frac{1}{4}(A_\mu \cdot A_\nu) \cdot (A_\mu \cdot A_\nu). \end{aligned} \quad (5)$$

To set the boundary conditions, one observes that the cutoff effective action $\Gamma[\Phi, \gamma; \Lambda, \Lambda_0]$ at $\Lambda = \Lambda_0$ becomes the bare action and to ensure perturbative renormalizability it must be local, thus irrelevant vertices must vanish at $\Lambda = \Lambda_0$. As far as the relevant couplings, the boundary conditions must provide the physical coupling $g(\mu)$ and guarantee symmetry. In fact the cutoff Λ explicitly breaks gauge invariance and one has to show that at the physical point $\Lambda = 0$ and $\Lambda_0 \rightarrow \infty$ the Slavnov-Taylor identity can be recovered. As shown in refs. [3, 4] the symmetry is ensured if the relevant couplings are properly fixed at some value of Λ . In the next section we briefly recall how this procedure can be implemented.

Once the boundary conditions are given, equation (3) can be thought as an alternative definition of the theory which in principle is non-perturbative. On the other hand the iterative solution of (3) reproduces the usual loop expansion and, as shown by Polchinski [2], the $\Lambda_0 \rightarrow \infty$ limit of $\Gamma[\Phi, \gamma; \Lambda, \Lambda_0]$ is finite for all values of Λ ¹. In the following this limit will be taken and $\Gamma[\Phi, \gamma; \Lambda, \infty]$ will be denoted by $\Gamma[\Phi, \gamma; \Lambda]$. Notice that, once this limit has been performed, the irrelevant vertices of $\Gamma[\Phi, \gamma; \Lambda]$ do not vanish at $\Lambda = \Lambda_0$ but are only suppressed by inverse powers of Λ_0 .

3 Effective ST identity

In the RG formulation the ST identity for the physical effective action $\Gamma[\Phi, \gamma] \equiv \Gamma[\Phi, \gamma; 0, \infty]$ is ensured if the cutoff effective action satisfies a modified ST [3]-[7]. As usual in studying

¹For a generalization of this proof to gauge theories see for instance [4].

ST identities, it is convenient to remove from the functional Π given in (4) the gauge fixing term and introduce the Slavnov operator [13]

$$\mathcal{S}_{\Pi'} = -i \int_p \left(\frac{\delta \Pi'}{\delta \Phi_i(-p)} \frac{\delta}{\delta \gamma_i(p)} + \frac{\delta \Pi'}{\delta \gamma_i(p)} \frac{\delta}{\delta \Phi_i(-p)} \right), \quad \Phi_i = \{A_\mu, c\}$$

with

$$\Pi'[\Phi, \gamma; \Lambda] = \Pi[\Phi, \gamma; \Lambda] + \frac{1}{2} \int_p p_\mu p_\nu A_\mu(-p) \cdot A_\nu(p).$$

The modified ST identity then reads

$$\Delta_\Gamma(\Lambda) \equiv \mathcal{S}_{\Pi'} \Pi'[\Phi, \gamma; \Lambda] + \hat{\Delta}_\Gamma(\Lambda) = 0, \quad (6)$$

where

$$\begin{aligned} \hat{\Delta}_\Gamma[\Phi, \gamma; \Lambda] &= -\hbar \int_{p,q} K_{0\Lambda}(p) (-1)^{\delta_L} [D_{\Lambda\infty}^{-1}(-p)]_{Li} \left[\frac{\delta^2 \Gamma[\Phi, \gamma; \Lambda]}{\delta \Phi_L(-p) \delta \Phi_J(-q)} \right]^{-1} \\ &\quad \times \frac{\delta^2}{\delta \Phi_J(q) \delta \gamma_i(p)} \left(\Pi[\Phi, \gamma; \Lambda] - \frac{1}{g} \int_x u_\mu \partial_\mu c \right). \end{aligned} \quad (7)$$

The factor \hbar has been inserted to put into evidence that $\hat{\Delta}_\Gamma$ vanishes at tree-level. Notice that at the physical point $\Lambda = 0$ the gauge symmetry condition (6) reduces to the usual ST identity, since Π becomes the physical effective action and $\hat{\Delta}_\Gamma$ vanishes. It can be shown that if Δ_Γ vanishes up to loop ℓ then at the next loop Δ_Γ is Λ -independent and therefore can be analyzed at every value of Λ . If one imposes (6) at $\Lambda = 0$, one gets the boundary conditions for the relevant part of the effective action given in terms of the physical coupling $g(\mu)$. Alternatively, in (6) one can choose Λ much bigger than all external momenta, namely $\Lambda = \Lambda_0$, and determine the cutoff dependent couplings of UV action. Both these procedures provide the boundary conditions of the RG flow for the non-invariant relevant couplings of the cutoff effective action. We will adopt the latter possibility in the present paper.

At $\Lambda = \Lambda_0$ the functional Δ_Γ is local, or more precisely, its irrelevant contributions vanish as inverse powers of Λ_0 and disappear in the $\Lambda_0 \rightarrow \infty$ limit (see [6] for a proof). This is obviously true for the functional $\mathcal{S}_{\Pi'} \Pi'(\Lambda_0)$, due to the boundary conditions imposed on the irrelevant vertices of Π , and can be shown by a direct calculation for the functional $\hat{\Delta}_\Gamma(\Lambda_0)$. Therefore in the $\Lambda_0 \rightarrow \infty$ limit the effective ST identity (6) reduces to a finite set of equations (the so called fine-tuning equations) obtained by requiring the vanishing of relevant part of Δ_Γ . In fact this system of equations overdetermines the UV couplings and in order to prove its solvability one must exploit the so-called consistency conditions which reduce the number of independent equations [3]. In the next section we will solve these fine-tuning equations at one-loop order tuning the couplings of the UV action at this order. Besides determining these couplings this calculation provides a direct check of the solvability of the fine-tuning problem.

4 One-loop computations

At one-loop and at $\Lambda = \Lambda_0$, the fine-tuning equation (6) reduces to

$$\mathcal{S}_{\Pi^{(0)}} \Pi^{(1)}(\Lambda_0) = -\hat{\Delta}_\Gamma^{(1)}(\Lambda_0), \quad (8)$$

where the local functional $\Pi^{(1)}(\Lambda_0) \equiv \Pi^{(1)}[\Phi, \gamma; \Lambda_0]$ contains the relevant monomials given in (5) (as discussed in the previous section its irrelevant contributions vanish as $\Lambda_0 \rightarrow \infty$ and therefore can be omitted in the calculations performed in this section). This functional can be split into two contributions ²

$$\Pi(\Lambda_0) = \Pi_{\text{inv}}(\Lambda_0) + \tilde{\Pi}(\Lambda_0), \quad (9)$$

where Π_{inv} satisfies the ST identity *i.e.* $\mathcal{S}_{\Pi_{\text{inv}}} \Pi_{\text{inv}} = 0$. The explicit form of Π_{inv} is

$$\Pi_{\text{inv}}(\Lambda_0) = \int d^4x \left\{ -\frac{1}{4} z_1 \mathcal{F}_{\mu\nu} \cdot \mathcal{F}^{\mu\nu} + z_2 z_3 \left(\frac{1}{g z_3} w_\mu \cdot \mathcal{D}_\mu c - \frac{1}{2} v \cdot c \wedge c \right) \right\},$$

with $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g z_3 A_\mu \wedge A_\nu$ and the covariant derivative given by $\mathcal{D}_\mu c = \partial_\mu c + g z_3 A_\mu \wedge c$. The remaining monomials contribute to $\tilde{\Pi}$ which can be written as

$$\begin{aligned} \tilde{\Pi}(\Lambda_0) \equiv & \int d^4x \left\{ \sigma_1 \Lambda_0^2 \frac{1}{2} A_\mu^2 + \sigma_2 \frac{1}{2} (\partial_\mu A_\mu)^2 + \sigma_3 w_\mu \cdot c \wedge A_\mu + \sigma_4 \frac{1}{2} v \cdot c \wedge c \right. \\ & \left. + \sigma_5 \frac{g^2}{4} (A_\mu \wedge A_\nu)^2 + \sigma_6 \frac{g^2}{4} (A_\mu \cdot A_\nu)^2 \right\}. \end{aligned} \quad (10)$$

The functional $\Pi(\Lambda_0)$ is equal to the UV action $S_{\Lambda_0}[\Phi, \gamma]$ with the UV wave function constants and UV couplings given by

$$z_i = z_i(g, \mu/\Lambda_0), \quad \sigma_i = \sigma_i(g, \mu/\Lambda_0).$$

Notice that the fine-tuning equation (8) allows to compute the couplings in $\Pi^{(1)}(\Lambda_0)$ since $\hat{\Delta}_\Gamma^{(1)}(\Lambda_0)$ depends only on $\Pi^{(0)} = S_{BRS}$. As a matter of fact only the couplings σ_i are tuned with this procedure. On the contrary the couplings z_i are not involved in the fine-tuning. They are free parameters which can be fixed setting the field normalization and the gauge coupling at the subtraction point μ equal to their physical values at $\Lambda = 0$, *i.e.* $z_i(\Lambda = 0) = 1$. In the standard language this corresponds to the renormalization prescriptions. In the following subsections we will determine the expression for the couplings σ_i in term of the Yang-Mills coupling g and the cutoff function. The values of these couplings corresponding to three different choices of the cutoff function are collected in the tables 1-3 given below.

Using the parameterization given in (9) and (10), the l.h.s. of (8) becomes

$$\begin{aligned} \mathcal{S}_{\Pi^{(0)}} \Pi^{(1)}(\Lambda_0) = & \int d^4x \left\{ \delta_1 \Lambda_0^2 A_\mu \cdot \partial_\mu c + \delta_2 A_\mu \cdot \partial^2 \partial_\mu c + \delta_3 A_\mu \cdot (\partial^2 A_\mu) \wedge c \right. \\ & + \delta_4 A_\mu \cdot (\partial_\mu \partial_\nu A_\nu) \wedge c + \frac{1}{2} \delta_5 (\partial_\mu w_\mu) \cdot c \wedge c + \frac{1}{2} \delta_6 (w_\mu \wedge A_\mu) \cdot (c \wedge c) \\ & + \delta_7 ((\partial_\mu A_\mu) \cdot A_\nu) (A_\nu \cdot c) + \delta_8 ((\partial_\mu A_\mu) \cdot c) (A_\nu \cdot A_\nu) + \delta_9 ((\partial_\nu A_\mu) \cdot A_\nu) (A_\mu \cdot c) \\ & \left. + \delta_{10} ((\partial_\nu A_\mu) \cdot A_\mu) (A_\nu \cdot c) + \delta_{11} ((\partial_\nu A_\mu) \cdot c) (A_\mu \cdot A_\nu) \right\} \end{aligned}$$

where

$$\begin{aligned} \delta_1 &= -\frac{i}{g} \sigma_1, & \delta_2 &= \frac{i}{g} \sigma_2, \\ \delta_3 &= -i \sigma_3, & \delta_4 &= -i(\sigma_2 - \sigma_3), \\ \delta_5 &= \frac{1}{g} \delta_6 = \frac{i}{g} (\sigma_3 - \sigma_4), \\ \delta_7 &= \delta_9 = \delta_{11} = ig(\sigma_3 + \sigma_6 - \sigma_5), & \delta_8 &= \frac{1}{2} \delta_{10} = ig(\sigma_5 - \sigma_3). \end{aligned} \quad (11)$$

²For the sake of simplicity the loop order index will be sometimes understood.

The system of equations obtained by imposing δ_i to be equal to the coefficient of the corresponding monomials in $-\hat{\Delta}_\Gamma$ overdetermines the couplings σ_i (eleven equations and five unknown couplings) therefore it can be solved if one has only five independent equations. The necessary six relations among the parameters δ_i are automatically obtained from the anticommutativity of the BRS operator \mathcal{S}_Π , as can be seen in (11). Though it is not evident that the same relations hold for the coefficients of $\hat{\Delta}_\Gamma$, the direct calculation given in the following subsections will show that this is true.

At one-loop order $\hat{\Delta}_\Gamma$ can be evaluated taking the integrand in (7) at tree level and noticing that $\Pi^{(0)} = S_{BRS}$ and then the sum over Φ_J is restricted to the (A, c) -fields. At $\Lambda = \Lambda_0$, $\hat{\Delta}_\Gamma^{(1)}$ is given by

$$\hat{\Delta}_\Gamma^{(1)}(\Lambda_0) = \int_{p,q} K_{0\Lambda_0}(p)(-)^{\delta_i} \left(D_{\Lambda_0\infty}(q) \bar{\Gamma}^{(0)}[-q, -p; \Lambda_0] \right)_{ji} \frac{\delta^2}{\delta\Phi_j(q)\delta\gamma_i(p)} \left(S_{BRS} - \frac{1}{g} \int_x u_\mu \partial_\mu c \right), \quad (12)$$

where the functional $\bar{\Gamma}^{(0)}$ originating from the inversion of $\frac{\delta^2\Gamma[\Phi, \gamma; \Lambda]}{\delta\Phi(-q)\delta\Phi(-p)}$ is recursively defined as

$$\begin{aligned} \bar{\Gamma}_{IJ}^{(0)}[-q, -p; \Phi, \gamma; \Lambda_0] &= (-)^{\delta_J} \left(S_{BRS}^{int}(-q, -p) \right)_{IJ} \\ &\quad - \int_{q'} \left(D_{\Lambda_0\infty}(q') \bar{\Gamma}^{(0)}[-q', -p; \Phi, \gamma; \Lambda_0] \right)_{LJ} \left(S_{BRS}^{int}(-q, q') \right)_{IL}, \end{aligned} \quad (13)$$

with S_{BRS}^{int} the interaction part of S_{BRS} and

$$\left(S_{BRS}^{int}(p, q) \right)_{IJ} = \frac{\delta}{\delta\Phi_J(q)} \frac{\delta}{\delta\Phi_I(p)} S_{BRS}^{int}.$$

For instance when Φ_I and Φ_J are vector fields one has

$$\left(S_{BRS}^{int}(p, q) \right)_{A_\nu^b A_\mu^a} = -ig\epsilon^{abc} t_{\mu\nu\rho}(p, q) A_\rho^c(-p-q) - g^2 t_{\mu\nu\rho\sigma}^{abcd} \int_k A_\rho^c(k) A_\sigma^d(-p-q-k), \quad (14)$$

where

$$t_{\mu\nu\rho}(p, q) = (p-q)_\rho g_{\mu\nu} + (2q+p)_\mu g_{\nu\rho} - (2p+q)_\nu g_{\mu\rho}$$

and

$$\begin{aligned} t_{\mu_1 \dots \mu_4}^{a_1 \dots a_4} &= (\epsilon^{a_1 a_2 c} \epsilon^{c a_3 a_4} - \epsilon^{a_1 a_4 c} \epsilon^{c a_2 a_3}) g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} + (\epsilon^{a_1 a_3 c} \epsilon^{c a_2 a_4} - \epsilon^{a_1 a_4 c} \epsilon^{c a_3 a_2}) g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} \\ &\quad + (\epsilon^{a_1 a_3 c} \epsilon^{c a_4 a_2} - \epsilon^{a_1 a_2 c} \epsilon^{c a_3 a_4}) g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} \end{aligned}$$

are the three and four-vector elementary vertices, respectively.

In the following subsections we compute the coefficients of the various field monomials in $\hat{\Delta}_\Gamma$.

4.1 Ac-vertex

There are three contributions³ to the $A_\mu \cdot c$ -vertex which are obtained inserting in (12) the first term of (13) and setting $(\gamma_i = w_\nu, \Phi_j = A_\nu)$, $(\gamma_i = w_\nu, \Phi_j = c)$ and $(\gamma_i = v, \Phi_j = c)$.

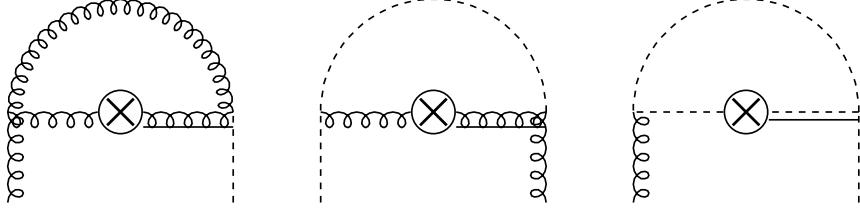


Figure 1: Graphical contribution to the $A\text{-}c$ -vertex of $\hat{\Delta}_\Gamma$. The curly and dashed line denotes the gluon and ghost field respectively; the double lines represent the BRS source associated to the field depicted by the top line. All momenta are incoming.

The corresponding graphs are given in Fig 1.

Using the vertices of S_{BRS} one obtains

$$\int_p A_\mu(p) \cdot c(-p) \hat{\Delta}_{\Gamma\mu}^{(Ac)}(p; \Lambda_0)$$

with

$$\hat{\Delta}_{\Gamma\mu}^{(Ac)}(p; \Lambda_0) = -2ig \int_q \left(t_{\nu\mu\nu}(q, p) - 2q_\mu \right) \frac{K_{\Lambda_0\infty}(q)}{q^2} K_{0\Lambda_0}(q + p).$$

The presence of the two cutoff functions having almost non-intersecting supports (*i.e.* $q^2 \gtrsim \Lambda_0^2$, $(q + p)^2 \lesssim \Lambda_0^2$) ensures that this vertex is quadratically divergent in the $\Lambda_0 \rightarrow \infty$ limit but without logarithmic divergences. In this limit one gets

$$\hat{\Delta}_{\Gamma\mu}^{(Ac)}(p; \Lambda_0) = i p_\mu [\hat{\delta}_1 \Lambda_0^2 - \hat{\delta}_2 p^2 + \mathcal{O}(p^4/\Lambda_0^2)]$$

with

$$\hat{\delta}_1 = \frac{2g}{\Lambda_0^2} \int_q \frac{(1 - K)(3K + 2K'q^2)}{q^2}, \quad (15)$$

and

$$\hat{\delta}_2 = -2g \int_q \frac{(1 - K)(18K' + 21K''q^2 + 4K'''q^4)}{6q^2}, \quad (16)$$

where $K \equiv K_{0\Lambda_0}(q)$ and the ' denotes the derivative with respect to q^2 and we have used the identity $K_{\Lambda_0\infty}(q) = 1 - K_{0\Lambda_0}(q)$. The coefficients $\hat{\delta}_1$ and $\hat{\delta}_2$ are finite and cutoff function dependent. Recalling that $K_{0\Lambda_0}(0) = 1$ and that $K_{0\Lambda_0}(q) \rightarrow 0$ rapidly enough for $q^2 \rightarrow \infty$, (15) and (16) can be rewritten as

$$\hat{\delta}_1 = \frac{2g}{\Lambda_0^2} \int_q \frac{(1 - 2K)K}{q^2} \quad (17)$$

and

$$\hat{\delta}_2 = g \left(\frac{5r}{6} - 3 \int_q K'^2 \right), \quad (18)$$

³This computation can be found in [7] and is repeated here for completeness.

where $r = i/(16\pi^2)$ (the factor i comes from the q -integration). The fine-tuning equation (8) for the Ac -vertex becomes

$$\delta_1 = -\hat{\delta}_1, \quad \delta_2 = -\hat{\delta}_2.$$

Then from (11) one finds that σ_1 and σ_2 are given by

$$\sigma_1^{(1)} = -\frac{2ig^2}{\Lambda_0^2} \int_q \frac{(1-2K)K}{q^2}, \quad (19)$$

$$\sigma_2^{(1)} = ig^2 \left(\frac{5r}{6} - 3 \int_q K'^2 \right) \quad (20)$$

and their values depend, as shown in table 1, on the cutoff function.

$K_{0\Lambda_0}(p)$	e^{-p^2/Λ_0^2}	$\frac{\Lambda_0^4}{(p^2+\Lambda_0^2)^2}$	$\frac{\Lambda_0^{2n}}{p^{2n}+\Lambda_0^{2n}}$
σ_1	0	$-\frac{2ig^2 r}{3}$	$\frac{2\pi i g^2 r(n-2)}{n^2 \sin(\pi/n)}$
σ_2	$\frac{ig^2 r}{12}$	$\frac{7ig^2 r}{30}$	$\frac{ig^2 r(5-3n)}{6}$

Table 1: One-loop values of the couplings σ_1 and σ_2 for three different cutoff functions ($r = \frac{i}{16\pi^2}$ and $n \geq 1$).

4.2 AAc -vertex

In order to find the AAc -vertex one has to insert in (12) the functional $\bar{\Gamma}^{(0)}$ obtained from the first iteration in (13), *i.e.*

$$-(-)^{\delta_J} \int_{q'} \left(D_{\Lambda_0\infty}(q') S_{BRS}^{int}(-q', -p) \right)_{LJ} \left(S_{BRS}^{int}(-q, q') \right)_{IL}. \quad (21)$$

One finds one contribution for ($\gamma_i = w_\nu, \Phi_j = A_\nu$), two for ($\gamma_i = w_\nu, \Phi_j = c$) and one for ($\gamma_i = v, \Phi_j = c$) (plus the terms obtained from the permutation of the two vectors). The corresponding graphs are given in Fig 2. Using the vertices of S_{BRS} one obtains

$$\frac{1}{2} \int_{p_1 p_2} A_{\mu_1}(p_1) \cdot A_{\mu_2}(p_2) \wedge c(-p_1 - p_2) \hat{\Delta}_{\Gamma \mu_1 \mu_2}^{(AAc)}(p_1, p_2; \Lambda_0)$$

with

$$\hat{\Delta}_{\Gamma \mu_1 \mu_2}^{(AAc)}(p_1, p_2; \Lambda_0)$$

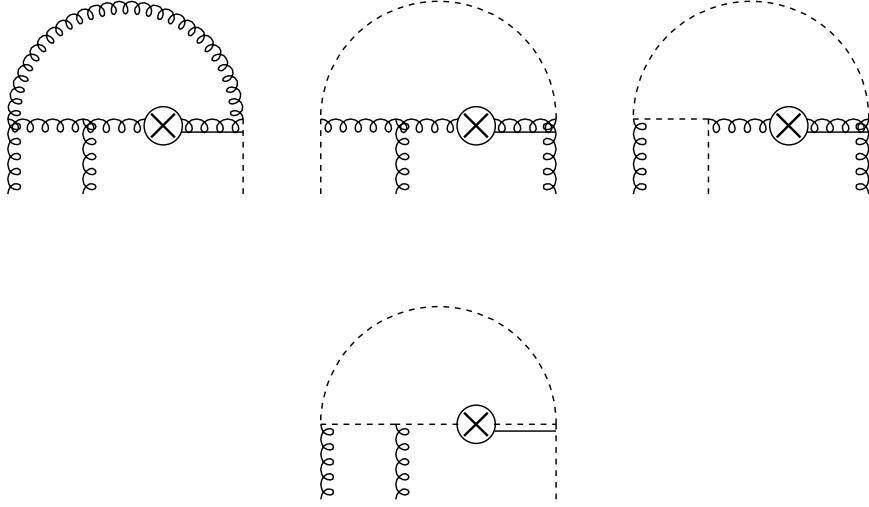


Figure 2: Graphical contribution to the A - A - c -vertex of $\hat{\Delta}_\Gamma$.

$$\begin{aligned}
&= -\frac{g^2}{2} \int_q \left[t_{\rho\mu_1\nu}(q, p_1) t_{\nu\mu_2\rho}(q + p_1, p_2) \frac{K_{\Lambda_0\infty}(q) K_{\Lambda_0\infty}(q + p_1)}{q^2(q + p_1)^2} K_{0\Lambda_0}(q + p_1 + p_2) \right. \\
&\quad - (q + p_1 + p_2)_\nu t_{\nu\mu_1\mu_2}(q, p_1) \frac{K_{\Lambda_0\infty}(q) K_{\Lambda_0\infty}(q + p_1 + p_2)}{q^2(q + p_1 + p_2)^2} K_{0\Lambda_0}(q + p_1) \\
&\quad - q_{\mu_1}(q + p_1)_{\mu_2} \frac{K_{\Lambda_0\infty}(q) K_{\Lambda_0\infty}(q + p_1)}{q^2(q + p_1)^2} (K_{0\Lambda_0}(q - p_2) + K_{0\Lambda_0}(q + p_1 + p_2)) \\
&\quad \left. -(1 \leftrightarrow 2) \right].
\end{aligned}$$

In the $\Lambda_0 \rightarrow \infty$ limit one gets

$$\hat{\Delta}_{\Gamma\mu_1\mu_2}^{(AAc)}(p_1, p_2; \Lambda_0) = g_{\mu_1\mu_2}(p_1^2 - p_2^2) \hat{\delta}_3 + (p_{1\mu_1} p_{1\mu_2} - p_{2\mu_1} p_{2\mu_2}) \hat{\delta}_4 + \mathcal{O}(P^4/\Lambda_0^2),$$

where P is a combination of the external momenta p_1 and p_2 ,

$$\hat{\delta}_3 = -g^2 \int_q \frac{(1-K)}{6q^4} [13K^2 - 2q^2(11K' + 8K'^2q^2 + K''q^2) + K(-13 + 15K'q^2 - 8K''q^4)] \quad (22)$$

and

$$\hat{\delta}_4 = g^2 \int_q \frac{(1-K)}{6q^4} [13K^2 + 2q^2(-8K' + 10K'^2q^2 - K''q^2) + K(-13 + 45K'q^2 + 10K''q^4)]. \quad (23)$$

The fine-tuning equation (8) for the AAc -vertex allows to find the value of σ_3 in term of g and the cutoff function. From $\delta_3 = \hat{\delta}_3$ and (11) one finds

$$\sigma_3^{(1)} = -ig^2 \left(\frac{37r}{36} - \int_q \frac{13K(1-K)^2 + 10q^4K'^2}{6q^4} \right) \quad (24)$$

$K_{0\Lambda_0}(p)$	e^{-p^2/Λ_0^2}	$\frac{\Lambda_0^4}{(p^2+\Lambda_0^2)^2}$	$\frac{\Lambda_0^{2n}}{p^{2n}+\Lambda_0^{2n}}$
σ_3	$-\frac{ig^2r(39\ln\frac{4}{3}+11)}{18}$	$\frac{49igr}{360}$	$-\frac{igr(10n^2-37n+39)}{36n}$

Table 2: One-loop values of the coupling σ_3 for three different cutoff functions ($r = \frac{i}{16\pi^2}$ and $n \geq 1$).

and its value for different choices of the cutoff function is given in table 2.

In order to check the consistency of our computation the fine-tuning equation $\delta_4 \equiv -i(\sigma_2 - \sigma_3) = \hat{\delta}_4$ must be automatically satisfied with the couplings σ_2 and σ_3 previously determined and given in (20) and (24), respectively. Alternatively one has to prove the consistency condition $\hat{\delta}_3 + \hat{\delta}_4 = -g\hat{\delta}_2$. From (22) and (23) one finds

$$-\frac{1}{g}(\hat{\delta}_4 + \hat{\delta}_3) = -g \int_q \frac{1-K}{q^2} (K' + 5KK' + 6K'^2q^2 + 3KK''q^2)$$

which becomes equal to (18) after integration by parts.

4.3 $AAAc$ -vertex

The $AAAc$ -vertex receives contribution from (21) and the term originating from the second iteration of (13) *i.e.*

$$(-)^{\delta_J} \int_{q'q''} \left(D_{\Lambda_0\infty}(q'') S_{BRS}^{int}(-q'', -p) \right)_{HJ} \left(D_{\Lambda_0\infty}(q') S_{BRS}^{int}(-q', q'') \right)_{LH} \left(S_{BRS}^{int}(-q, q') \right)_{IL}. \quad (25)$$

The corresponding graphs are shown in figs. 5-7 and the various contributions are computed in Appendix A. After performing the trace over the gauge indices this vertex can be written as

$$\frac{1}{3!} \int_{p_1 p_2 p_3} \left(A_{\mu_1}(p_1) \cdot A_{\mu_2}(p_2) \right) \left(A_{\mu_3}(p_3) \cdot c(-p_1 - p_2 - p_3) \right) \hat{\Delta}_{\Gamma \mu_1 \mu_2 \mu_3}^{(AAAc)}(p_1, p_2, p_3; \Lambda_0).$$

In the $\Lambda_0 \rightarrow \infty$ limit one obtains

$$\begin{aligned} \hat{\Delta}_{\Gamma \mu_1 \mu_2 \mu_3}^{(AAAc)}(p_1, p_2, p_3; \Lambda_0) = & -3i[\hat{\delta}_7(p_{1\mu_1}g_{\mu_2\mu_3} + p_{2\mu_2}g_{\mu_1\mu_3}) + 2\hat{\delta}_8p_{3\mu_3}g_{\mu_1\mu_2} \\ & + \hat{\delta}_9(p_{1\mu_2}g_{\mu_1\mu_3} + p_{2\mu_1}g_{\mu_2\mu_3}) + \hat{\delta}_{10}(p_{1\mu_3} + p_{2\mu_3})g_{\mu_1\mu_2} + \hat{\delta}_{11}(p_{3\mu_2}g_{\mu_1\mu_3} + p_{3\mu_1}g_{\mu_2\mu_3})] \\ & + \mathcal{O}(P^4/\Lambda_0^2), \end{aligned} \quad (26)$$

where

$$\hat{\delta}_7 = \hat{\delta}_9 = \hat{\delta}_{11} = -g^3 \int_q \frac{(1-K)}{6q^4} [31K^3 + K^2(124K'q^2 - 47) + K(16 - 111K'q^2) + 14K'q^2] \quad (27)$$

and

$$\hat{\delta}_8 = \frac{1}{2} \hat{\delta}_{10} = -g^3 \int_q \frac{(1-K)}{6q^4} [23K^3 + K^2(56K'q^2 - 25) + K(2 - 48K'q^2) + K'q^2]. \quad (28)$$

By using the fine-tuning equation (8), together with the results (11) and (24), the couplings σ_5 and σ_6 are given by

$$\begin{aligned} \sigma_5^{(1)} &= -ig^2 \left(\frac{3r}{2} + \int_q \frac{K(1-K)(23K^2 - 12K - 11) - 10K'^2q^4}{6q^4} \right), \\ \sigma_6^{(1)} &= -ig^2 \left(\frac{2r}{3} + \int_q \frac{(1-K)(9K^3 + 12K^2 + 3K)}{q^4} \right) \end{aligned}$$

and their values for different cutoff functions are given in table 3.

$K_{0\Lambda_0}(p)$	e^{-p^2/Λ_0^2}	$\frac{\Lambda_0^4}{(p^2+\Lambda_0^2)^2}$	$\frac{\Lambda_0^{2n}}{p^{2n}+\Lambda_0^{2n}}$
σ_5	$-\frac{ig^2r(13+94\ln 2-70\ln 3)}{12}$	$\frac{47ig^2r}{630}$	$\frac{ig^2(5n^2-27n+28)}{18n}$
σ_6	$-\frac{ig^2r(2-63\ln 3+99\ln 2)}{3}$	$-\frac{58ig^2r}{105}$	$-\frac{2ig^2}{3}$

Table 3: One-loop values of the couplings σ_5 and σ_6 for three different cutoff functions ($r = \frac{i}{16\pi^2}$ and $n \geq 1$).

4.4 wcc-vertex

The *wcc*-vertex is obtained inserting (21) in (12) and is given by the three graphs depicted in Fig. 3. Using the vertices of S_{BRS} one gets

$$\frac{1}{2} \int_{p_1 p_2} w_\mu(-p_1 - p_2) \cdot c(p_1) \wedge c(p_2) \hat{\Delta}_{\Gamma \mu}^{(wcc)}(p_1, p_2; \Lambda_0)$$

with

$$\begin{aligned} &\hat{\Delta}_{\Gamma \mu}^{(wcc)}(p_1, p_2; \Lambda_0) \\ &= g \int_q \left[(q - p_1 - p_2)_\mu \frac{K_{\Lambda_0 \infty}(q) K_{\Lambda_0 \infty}(q - p_1 - p_2) K_{0\Lambda_0}(q - p_2)}{q^2(q - p_1 - p_2)^2} \right. \\ &\quad \left. - q_\mu \frac{K_{\Lambda_0 \infty}(q) K_{\Lambda_0 \infty}(q + p_2) [K_{0\Lambda_0}(q + p_1 + p_2) + K_{0\Lambda_0}(q - p_1)]}{q^2(q + p_2)^2} \right. \\ &\quad \left. + (1 \leftrightarrow 2) \right]. \end{aligned}$$

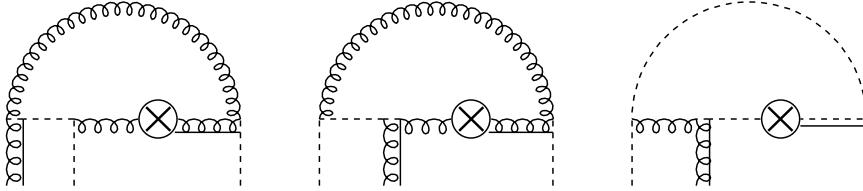


Figure 3: Graphical contribution to the $w\text{-}c\text{-}c$ -vertex of $\hat{\Delta}_\Gamma$.

In the $\Lambda_0 \rightarrow \infty$ limit one obtains

$$\hat{\Delta}_{\Gamma\mu}^{(wcc)}(p_1, p_2; \Lambda_0) = i(p_1 + p_2)_\mu [\hat{\delta}_5 + \mathcal{O}(P^2/\Lambda_0^2)],$$

where

$$\hat{\delta}_5 = g \int_q \frac{(-1 + 4K - 3K^2)K'}{q^2} = 0$$

as integration by parts can show. Thus from (11) the one-loop value of the coupling σ_4 is given by (24).

4.5 $wccA$ -vertex

The fine-tuning equation for this vertex is directly obtained from the relation between δ_5 and δ_6 in (11), thus this vertex must vanish.

The graphs which contribute to this vertex are obtained inserting (25) in (12) and are given in fig. 4. All the terms can be collected in the following monomial

$$\int_{p_1 p_2 p_3} (A_\mu(p_3) \cdot c(p_2)) (w_\nu(-p_1 - p_2 - p_3) \cdot c(p_1)) \hat{\Delta}_{\Gamma\mu\nu}^{(Accw)}(p_3, p_2, p_1, p_3; \Lambda_0).$$

The complete expression of $\hat{\Delta}_{\Gamma\mu\nu}^{(Accw)}$ is not needed since

$$\hat{\Delta}_{\Gamma\mu\nu}^{(Accw)}(p_3, p_2, p_1, p_3; \Lambda_0) = g_{\mu\nu} \hat{\delta}_6 + \mathcal{O}(P^2/\Lambda_0^2)$$

and therefore in the $\Lambda_0 \rightarrow \infty$ limit one can set to zero all the external momenta. In this limit one obtains

$$g_{\mu\nu} \hat{\delta}_6 = g^2 \int_q \frac{(1-K)^3 K}{q^6} [-q^2 g_{\mu\nu} + q_\mu q_\nu + q_\rho t_{\rho\nu\mu}(q, 0)],$$

which vanishes.

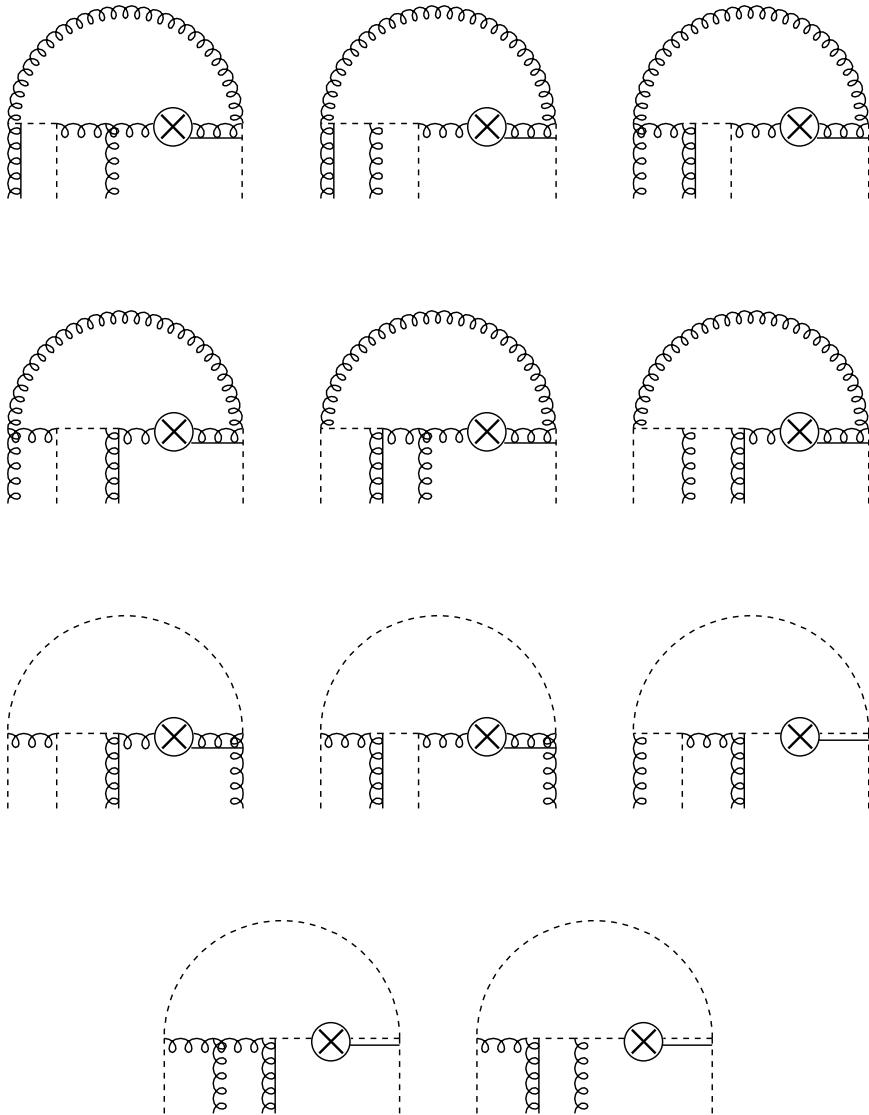


Figure 4: Graphical contribution to the $w\text{-}c\text{-}c\text{-}A$ -vertex of $\hat{\Delta}_\Gamma$.

5 Conclusions

The computation presented in this paper has shown that the requirement that the physical effective action satisfies the ST identity translates into a fine-tuning equation which allows to determine the couplings of the relevant, non-invariant interactions in the bare action $\Pi(\Lambda_0)$. Once the field normalization and the gauge coupling are fixed at the physical point $\Lambda = 0$, the problem of assigning the boundary conditions of the RG flow is finally solved. We explicitly compute the non-invariant couplings at one-loop, they turns out to be finite. This procedure systematically generalizes to higher orders. There are actually two crucial points which deserve mentioning. First of all one must discuss the locality of $\hat{\Delta}_\Gamma$. At one-loop order this has been explicitly shown in section 4 and is ensured by the fact that at tree level and at the UV scale the functional $(\frac{\delta^2 \Gamma}{\delta \Phi \delta \Phi})^{-1}$ is given by either a relevant vertex or a sequence of relevant vertices joint by propagators with a cutoff function $K_{\Lambda_0 \infty}(q + P)$, where P is a combination of external momenta. Since the integral in (7) is damped by these cutoff functions, only the contributions with a restricted number of propagators survive in the $\Lambda_0 \rightarrow \infty$ limit. One can infer from power counting that they are of the relevant type. A similar argument holds for the possible non-local contributions coming from $K_{0\Lambda_0}(p)$. Nevertheless at higher loops one could worry about non-local terms which may arise from the full propagators, $\Pi(\Lambda_0)$ and the proper vertices generated in the expansion of $(\frac{\delta^2 \Gamma}{\delta \Phi \delta \Phi})^{-1}$. With a little thought, it is easy to realize that the terms of this expansion which survive in the $\Lambda_0 \rightarrow \infty$ limit have less than three propagators, *i.e.* one more than in the one-loop case. This related to the fact that at higher loops the $w\text{-}c$ -vertex in $\Pi(\Lambda_0)$ contributes to (7) and therefore all the fields of $\hat{\Delta}_\Gamma$ originate from the inversion of $\frac{\delta^2 \Gamma}{\delta \Phi \delta \Phi}$ (recall that the relevant part of the functional Δ_Γ contains at most four fields). For the same reason, irrelevant proper vertices with at most six legs have to be taken into account. By dimensional analysis non-local parts of these vertices are proportional to negative powers of Λ_0 , yet they can contribute to the relevant part of $\hat{\Delta}_\Gamma$ since in (7) the loop momentum q is of the order of Λ_0 . As a consequence higher derivative interactions (*i.e.* vertices with higher powers of the momenta, but a restricted number of fields), are also involved in the fine-tuning. In particular one has to consider irrelevant interactions involving the BRS sources. Therefore at higher loops the fine-tuning procedure is rather cumbersome but well defined since the irrelevant vertices are completely determined by the flow equation (3) while the relevant ones are given by the fine-tuning at the lower loop orders.

Another issue to discuss is the finiteness of the non-invariant couplings at higher loops. The presence in (7) of the cutoff functions $K_{0\Lambda_0}(q + P_i)$ and $K_{\Lambda_0 \infty}(q + P_j)$, having almost non-intersecting supports, makes the q -integration in (7) finite. Actually the cutoff function $K_{0\Lambda_0}(q)$ must fall off more rapidly than any negative power of q for $q^2 > \Lambda_0^2$, in order to compensate the powers of q in the irrelevant vertices. Divergent contributions to the non-invariant couplings σ_i can be found only when we consider in (7) the relevant vertices $z_i(\Lambda_0)$, but they can be re-absorbed in field and gauge coupling redefinition.

Clearly, more elegant formulations exist for a pure gauge theory, such as dimensional regularization with minimal subtraction, which preserves BRS invariance and thus avoids the fine-tuning. However the RG formulation is general and can be applied to chiral gauge theories without anomalies along the same lines. In this case all the regularization procedures break the gauge symmetry and the fine-tuning is unavoidable. Moreover in the

dimensional regularization with minimal subtraction scheme the breaking of the symmetry requires the introduction of all the possible non-invariant counterterms⁴ while in the RG approach only interactions which are invariant under global chiral transformation need to be considered in the fine-tuning procedure [7].

As a final remark we should mention that it has been suggested [15] to use gauge invariant variables, such as the Wilson loop, to overcome the difficulty of the fine-tuning. However is not obvious how this procedure can be extended to chiral gauge theories.

Acknowledgements We have greatly benefited from discussions with G. Marchesini and F. Vian.

Appendix A

In this appendix we compute the various contributions to the $AAAc$ -vertex. We first consider in (12) the terms with $\gamma_i = w_\mu$. In this case both the contributions to the functional $\bar{\Gamma}^{(0)}$ given in (21) and (25) are needed to compute this vertex. The former involves the four-vector vertex given in (14) and the corresponding graphs are shown in Fig. 5.

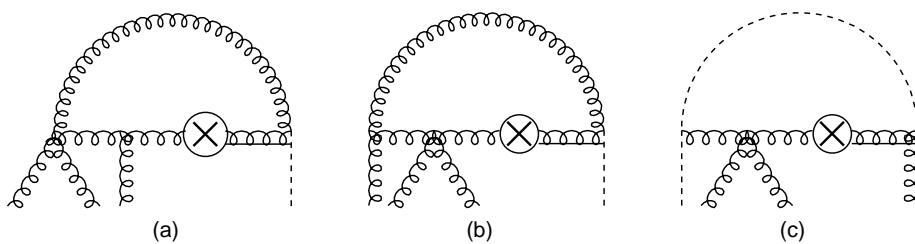


Figure 5: Graphical contribution to the A - A - A - c -vertex of $\hat{\Delta}_\Gamma$.

The contribution to $\hat{\Delta}_{\mu_1\mu_2\mu_3}^{(AAAc)}(p_1, p_2, p_3; \Lambda_0)$ from the graph in fig. 5a is (recall that p_1 , p_2 and p_3 are the momenta of the vector fields and that one has to consider the three permutations obtained by choosing the two external vectors which enters in the four-vector vertex)

$$3ig^3 \int_q \frac{K_{\Lambda_0\infty}(q)}{q^2} K_{0\Lambda_0}(q + p_1 + p_2 + p_3) \left\{ 2g_{\mu_1\mu_2}t_{\mu\mu_3\mu}(q + p_{12}, p_3) \frac{K_{\Lambda_0\infty}(q + p_{12})}{(q + p_{12})^2} \right. \\ \left. + g_{\mu_1\mu_3}t_{\mu\mu_2\mu}(q + p_{13}, p_2) \frac{K_{\Lambda_0\infty}(q + p_{13})}{(q + p_{13})^2} + g_{\mu_2\mu_3}t_{\mu\mu_1\mu}(q + p_{23}, p_1) \frac{K_{\Lambda_0\infty}(q + p_{23})}{(q + p_{23})^2} \right\}$$

⁴ Recently [14] these counterterms have been systematically computed at one-loop order in the dimensional regularization scheme.

$$\begin{aligned}
& + [t_{\mu_1\mu_2\mu_3}(q+p_{13}, p_2) - 2t_{\mu_3\mu_2\mu_1}(q+p_{13}, p_2)] \frac{K_{\Lambda_0\infty}(q+p_{13})}{(q+p_{13})^2} \\
& + [t_{\mu_2\mu_1\mu_3}(q+p_{23}, p_1) - 2t_{\mu_3\mu_1\mu_2}(q+p_{23}, p_1)] \frac{K_{\Lambda_0\infty}(q+p_{23})}{(q+p_{23})^2} \\
& - [t_{\mu_1\mu_3\mu_2}(q+p_{12}, p_3) + t_{\mu_2\mu_3\mu_1}(q+p_{12}, p_3)] \frac{K_{\Lambda_0\infty}(q+p_{12})}{(q+p_{12})^2} \Big\}.
\end{aligned}$$

For $\Lambda_0 \rightarrow \infty$ this graph gives the following contribution to the various coefficients $\hat{\delta}_i$ in (26)

$$\begin{aligned}
\hat{\delta}_7^{(5a)} &= \int_q \frac{(1-K)(7K^2 - 7K'q^2 - K(7 - 10K'q^2))}{2q^4} \\
\hat{\delta}_8^{(5a)} &= \int_q \frac{(1-K)(5K^2 - 5K'q^2 - K(5 - 6K'q^2))}{2q^4} \\
\hat{\delta}_9^{(5a)} &= - \int_q \frac{(1-K)(2K^2 + 7K'q^2 - K(2 + 13K'q^2))}{2q^4} \\
\hat{\delta}_{10}^{(5a)} &= \int_q \frac{(1-K)(19K^2 - 10K'q^2 - 19K(1 - K'q^2))}{2q^4} \\
\hat{\delta}_{11}^{(5a)} &= \int_q \frac{(1-K)(7K^2 - 7K'q^2 - K(7 - 12K'q^2))}{2q^4}.
\end{aligned}$$

The graph in Fig. 5b gives

$$\begin{aligned}
3ig^3 \int_q \frac{K_{\Lambda_0\infty}(q)}{q^2} K_{0\Lambda_0}(q+p_1+p_2+p_3) &\left\{ 2g_{\mu_1\mu_2}t_{\mu\mu_3\mu}(q, p_3) \frac{K_{\Lambda_0\infty}(q+p_3)}{(q+p_3)^2} \right. \\
& + g_{\mu_1\mu_3}t_{\mu\mu_2\mu}(q, p_2) \frac{K_{\Lambda_0\infty}(q+p_2)}{(q+p_2)^2} + g_{\mu_2\mu_3}t_{\mu\mu_1\mu}(q, p_1) \frac{K_{\Lambda_0\infty}(q+p_1)}{(q+p_1)^2} \\
& - [2t_{\mu_1\mu_2\mu_3}(q, p_2) - t_{\mu_3\mu_2\mu_1}(q, p_2)] \frac{K_{\Lambda_0\infty}(q+p_2)}{(q+p_2)^2} \\
& - [2t_{\mu_2\mu_1\mu_3}(q, p_1) - t_{\mu_3\mu_1\mu_2}(q, p_1)] \frac{K_{\Lambda_0\infty}(q+p_1)}{(q+p_1)^2} \\
& \left. - [t_{\mu_1\mu_3\mu_2}(q, p_3) + t_{\mu_2\mu_3\mu_1}(q, p_3)] \frac{K_{\Lambda_0\infty}(q+p_3)}{(q+p_3)^2} \right\}.
\end{aligned}$$

In the $\Lambda_0 \rightarrow \infty$ limit the various coefficients $\hat{\delta}_i$ for this term are given by

$$\begin{aligned}
\hat{\delta}_7^{(5b)} &= \int_q \frac{(1-K)(11K - 7)K'}{2q^4} \\
\hat{\delta}_8^{(5b)} &= \int_q \frac{(1-K)(9K - 5)K'}{2q^4} \\
\hat{\delta}_9^{(5b)} &= \int_q \frac{(1-K)(9K^2 - 9K - 7K'q^2 + 8KK'q^2)}{2q^4} \\
\hat{\delta}_{10}^{(5b)} &= - \int_q \frac{(1-K)(9K^2 - 9K + 10K'q^2 - 11KK'q^2)}{2q^4} \\
\hat{\delta}_{11}^{(5b)} &= \int_q \frac{(1-K)(9K - 7)K'}{2q^4}.
\end{aligned}$$

Finally from Fig. 5c we get

$$\begin{aligned} -3ig^3 \int_q \frac{K_{\Lambda_0\infty}(q)}{q^2} K_{0\Lambda_0}(q + p_1 + p_2 + p_3) & \left\{ \frac{K_{\Lambda_0\infty}(q-p_3)}{(q-p_3)^2} [2g_{\mu_1\mu_2}q_{\mu_3} - g_{\mu_2\mu_3}q_{\mu_1} - g_{\mu_1\mu_3}q_{\mu_2}] \right. \\ & + \frac{K_{\Lambda_0\infty}(q-p_2)}{(q-p_2)^2} [g_{\mu_1\mu_3}q_{\mu_2} + g_{\mu_1\mu_2}q_{\mu_3} - 2g_{\mu_2\mu_3}q_{\mu_1}] \\ & \left. + \frac{K_{\Lambda_0\infty}(q-p_1)}{(q-p_1)^2} [g_{\mu_2\mu_3}q_{\mu_1} + g_{\mu_1\mu_2}q_{\mu_3} - 2g_{\mu_1\mu_3}q_{\mu_2}] \right\}. \end{aligned}$$

The $\hat{\delta}_i$ for this graph are

$$\begin{aligned} \hat{\delta}_7^{(5c)} &= - \int_q \frac{(1-K)(2K^2 - 2K - K'q^2 - KK'q^2)}{2q^4} \\ \hat{\delta}_8^{(5c)} = -\hat{\delta}_{11}^{(5c)} &= \int_q \frac{(1-K)(2K^2 - 2K + K'q^2 - 3KK'q^2)}{2q^4} \\ \hat{\delta}_9^{(5c)} &= - \int_q \frac{(1-K)(K^2 - K + K'q^2 - 2KK'q^2)}{q^4} \\ \hat{\delta}_{10}^{(5c)} &= \int_q \frac{(1-K)(4K^2 - 4K + K'q^2 - 5KK'q^2)}{2q^4}. \end{aligned}$$

All the remaining contributions to $\hat{\Delta}_{\Gamma\mu_1\mu_2\mu_3}^{(AAAc)}(p_1, p_2, p_3; \Lambda_0)$ are generated from the part of the functional $\bar{\Gamma}^{(0)}$ given in (25). For $\gamma_i = w_\mu$ there are four different terms according to the position of the ghost field in (25) which are depicted in Fig. 6. The contribution from

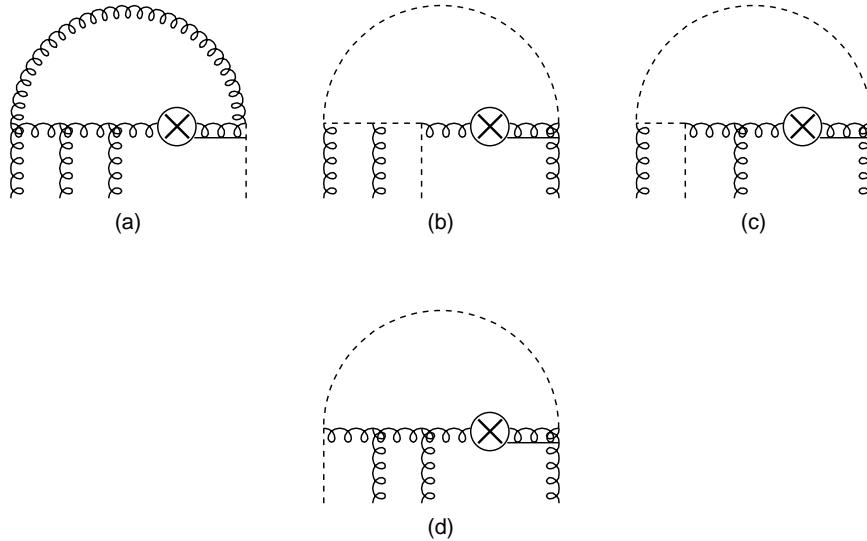


Figure 6: Graphical contribution to the A - A - A - c vertex of $\hat{\Delta}_\Gamma$.

the graph in Fig. 6a is

$$-ig^3 \int_q \frac{K_{\Lambda_0\infty}(q)}{q^2} K_{0\Lambda_0}(q + p_1 + p_2 + p_3)$$

$$\begin{aligned}
& \times \left\{ t_{\mu\mu_1\nu}(q, p_1) t_{\nu\mu_2\rho}(q + p_1, p_2) t_{\rho\mu_3\mu}(q + p_{12}, p_3) \frac{K_{\Lambda_0\infty}(q + p_1) K_{\Lambda_0\infty}(q + p_{12})}{(q + p_1)^2(q + p_{12})^2} \right. \\
& + t_{\mu\mu_3\nu}(q, p_3) t_{\nu\mu_2\rho}(q + p_3, p_2) t_{\rho\mu_1\mu}(q + p_{23}, p_1) \frac{K_{\Lambda_0\infty}(q + p_3) K_{\Lambda_0\infty}(q + p_{23})}{(q + p_3)^2(q + p_{23})^2} \Big\} \\
& + \text{permutations} ,
\end{aligned}$$

where the permutations are performed among the vector fields. For $\Lambda_0 \rightarrow \infty$ the contribution of this graph to the various coefficients $\hat{\delta}_i$ in (26) is

$$\begin{aligned}
\hat{\delta}_7^{(6a)} &= 2\hat{\delta}_8^{(6a)} = \hat{\delta}_9^{(6a)} = \hat{\delta}_{10}^{(6a)} = \hat{\delta}_{11}^{(6a)} = \\
&-5 \int_q \frac{(1-K)^2(K^2 - 2K'q^2 - K(1-4K'q^2))}{q^4} .
\end{aligned}$$

The graph in Fig. 6b gives

$$\begin{aligned}
ig^3 \int_q \frac{K_{\Lambda_0\infty}(q) K_{\Lambda_0\infty}(q + p_1)}{q^2(q + p_1)^2} q_{\mu_1} \left\{ (q + p_1)_{\mu_2} (q + p_{12})_{\mu_3} \frac{K_{\Lambda_0\infty}(q + p_{12})}{(q + p_{12})^2} K_{0\Lambda_0}(q - p_3) \right. \\
\left. + (q + p_1)_{\mu_3} (q + p_{13})_{\mu_2} \frac{K_{\Lambda_0\infty}(q + p_{13})}{(q + p_{13})^2} K_{0\Lambda_0}(q - p_2) \right\} + \text{permutations} .
\end{aligned}$$

The $\hat{\delta}_i$ for this graph are

$$\begin{aligned}
\hat{\delta}_7^{(6b)} &= \int_q \frac{(1-K)^2(5K^2 - K'q^2 - K(5+4K'q^2))}{12q^4} \\
\hat{\delta}_8^{(6b)} &= \int_q \frac{(1-K)^2(K^2 - K'q^2 - K)}{12q^4} \\
\hat{\delta}_9^{(6b)} &= - \int_q \frac{(1-K)^2(K^2 + K'q^2 - K(1-4K'q^2))}{12q^4} \\
\hat{\delta}_{10}^{(6b)} &= - \int_q \frac{(1-K)^2(4K^2 + K'q^2 - K(1-4K'q^2))}{12q^4} \\
\hat{\delta}_{11}^{(6b)} &= - \int_q \frac{(1-K)^2(K^2 + 2K'q^2 - K)}{12q^4} .
\end{aligned}$$

From Fig. 6c we get the contribution

$$\begin{aligned}
&-ig^3 \int_q \left\{ q_{\mu_3}(q + p_3)_\mu t_{\mu\mu_2\mu_1}(q - p_{12}, p_2) \frac{K_{\Lambda_0\infty}(q + p_3) K_{\Lambda_0\infty}(q - p_{12})}{(q + p_3)^2(q - p_{12})^2} K_{0\Lambda_0}(q - p_1) \right. \\
&+ q_{\mu_1}(q + p_1)_\mu t_{\mu\mu_3\mu_2}(q - p_{23}, p_3) \frac{K_{\Lambda_0\infty}(q + p_1) K_{\Lambda_0\infty}(q - p_{23})}{(q + p_1)^2(q - p_{23})^2} K_{0\Lambda_0}(q - p_2) \Big\} \\
&\times \frac{K_{\Lambda_0\infty}(q)}{q^2} + \text{permutations}
\end{aligned}$$

which gives

$$\hat{\delta}_7^{(6c)} = - \int_q \frac{(1-K)^2(4K^2 + K'q^2 - K(4+5K'q^2))}{6q^4}$$

$$\begin{aligned}\hat{\delta}_8^{(6c)} &= -\hat{\delta}_{11}^{(6c)} = \int_q \frac{(1-K)^2 K K'}{2q^4} \\ \hat{\delta}_9^{(6c)} &= - \int_q \frac{(1-K)^2 (5K^2 - 4K'q^2 - K(5 - 8K'q^2))}{12q^4} \\ \hat{\delta}_{10}^{(6c)} &= \int_q \frac{(1-K)^2 (19K^2 + 4K'q^2 - K(19 + 14K'q^2))}{12q^4}.\end{aligned}$$

From Fig. 6d one has

$$\begin{aligned}ig^3 \int_q q_\mu \left\{ t_{\mu\mu_1\nu}(q-p_1-p_2-p_3, p_1) t_{\nu\mu_2\mu_3}(q-p_{23}, p_2) \frac{K_{\Lambda_0\infty}(q-p_{23})}{(q-p_{23})^2} K_{0\Lambda_0}(q-p_3) \right. \\ \left. + t_{\mu\mu_3\nu}(q-p_1-p_2-p_3, p_3) t_{\nu\mu_2\mu_1}(q-p_{12}, p_2) \frac{K_{\Lambda_0\infty}(q-p_{12})}{(q-p_{12})^2} K_{0\Lambda_0}(q-p_1) \right\} \\ \times \frac{K_{\Lambda_0\infty}(q) K_{\Lambda_0\infty}(q-p_1-p_2-p_3)}{q^2(q-p_1-p_2-p_3)^2} + \text{permutations}\end{aligned}$$

and finds

$$\begin{aligned}\hat{\delta}_7^{(6d)} &= \int_q \frac{(1-K)^2 (5K^2 - 7K'q^2 - K(5 + 22K'q^2))}{12q^4} \\ \hat{\delta}_8^{(6d)} &= - \int_q \frac{(1-K)^2 (15K^2 + 7K'q^2 - K(15 + 22K'q^2))}{12q^4} \\ \hat{\delta}_9^{(6d)} &= \int_q \frac{(1-K)^2 (2K^2 + 11K'q^2 - 2K(1 + 14K'q^2))}{12q^4} \\ \hat{\delta}_{10}^{(6d)} &= - \int_q \frac{(1-K)^2 (49K^2 + K'q^2 - K(49 + 26K'q^2))}{12q^4} \\ \hat{\delta}_{11}^{(6d)} &= - \int_q \frac{(1-K)^2 (3K^2 - 10K'q^2 - K(3 - 34K'q^2))}{12q^4}.\end{aligned}$$

Finally there is a contribution to $\hat{\Delta}_{\Gamma\mu_1\mu_2\mu_3}^{(AAAc)}(p_1, p_2, p_3; \Lambda_0)$ for $\gamma_i = v$ which is depicted in Fig. 7 and gives

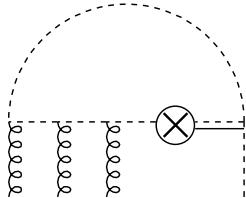


Figure 7: Graphical contribution to the A - A - A - c vertex of $\hat{\Delta}_{\Gamma}$.

$$\begin{aligned}
& -ig^3 \int_q \frac{K_{\Lambda_0\infty}(q)}{q^2} K_{0\Lambda_0}(q + p_1 + p_2 + p_3) \\
& \times \left\{ q_{\mu_1}(q + p_1)_{\mu_2}(q + p_{12})_{\mu_3} \frac{K_{\Lambda_0\infty}(q + p_1)K_{\Lambda_0\infty}(q + p_{12})}{(q + p_1)^2(q + p_{12})^2} \right. \\
& \left. + q_{\mu_3}(q + p_3)_{\mu_2}(q + p_{23})_{\mu_1} \frac{K_{\Lambda_0\infty}(q + p_3)K_{\Lambda_0\infty}(q + p_{23})}{(q + p_3)^2(q + p_{23})^2} \right\} \\
& + \text{permutations.}
\end{aligned}$$

The contribution to the $\hat{\delta}_i$ of this graph

$$\hat{\delta}_7^{(7)} = 2\hat{\delta}_8^{(7)} = \hat{\delta}_9^{(7)} = \hat{\delta}_{10}^{(7)} = \hat{\delta}_{11}^{(7)} = - \int_q \frac{(1-K)^2(K^2 + K'q^2 - K(1+2K'q^2))}{3q^4}.$$

After putting together all these results one obtains (27) and (28).

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